Applied Physics – Gears Math

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Basics Terms

A gear is a set of toothed wheels (gear wheels or cog-wheels) that work together to transmit movement.

Many of the everyday mechanisms and devices we commonly use contain gear wheels. These include bicycles, cars, and can-openers.

Gears are used to create these effects:
1. To change the position of a rotating movement. (This is sometimes called applying the rotation at a distance.)
2. To change the direction of rotation.
3. To increase or decrease speed of rotation.
4. To increase turning force (This is sometimes called torque.)

Key Words

**Driver/Input**
The name for a gear wheel that is turned by an outside force (such as that from a motor or from a person turning a handle) and that also turns at least one other gear wheel.

**Driven/Follower/Output**
The name for a gear wheel that is turned by another gear wheel.

**Gear Ratio**
A proportion used to compare how two meshed gear wheels move relative to each other. For gears, use the number of teeth for calculation. For pulleys, use its diameter for calculation.

**Gearing Down**
An arrangement in which a small driver turns a large follower, resulting in a slowing down of the turning. Gearing down produces a powerful turning force (torque).

**Gearing Up**
An arrangement in which a large driver turns a small follower, resulting in a speeding up of the turning. Gearing up reduces the turning force.

**Idler Gear**
The name for a gear wheel that is meshed between a driver and a follower. It does not mean it does not move. It is called idler gear because it does not affect the final gear ratio.

**Warm up information**
- Two meshed gear wheels turn in opposite directions.
- When two gear wheels are mounted on the same axle, they both turn at the same speed, regardless of their sizes.
- Gears have a trade-off with turning force (torque) and turning speed.
- In general, torque↑ speed ↓ when torque ↓ speed ↑
**Introduction to Simple Gears Transmission**

**Torque (twisting or turning force) is inversely proportional to speed.**

In order to determine both the speed and force of rotating axles, we need to calculate the **Gear Ratio**.

The gear ratio is the ratio of the number of teeth on each gear. Here is a gear with 8 teeth meshed with a gear with 40 teeth.

- **Gear ratio for this contraption is:**
  
  \[
  \frac{40}{8} = \frac{5}{1} \quad \text{or} \quad 5:1
  \]

What does this gear ratio \(\frac{5}{1}\) tell us?

- The Input/driver gear will rotate 5X when the output/follower gear rotates 1X
- The Input/driver gear will rotate 5X faster than the output/follower gear.
- This contraption is meant to increase torque
### Practice in Calculating Gear Ratio

**Provided:**
- Driving gear is on the right.
- Possible gear sizes are 40, 24, 12, 8 tooth gears.

**Required:**
- Calculate the ratio.
- Write “Driver turns N\(X\), Follower turns M\(X\)

<table>
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<tr>
<th>Provided:</th>
<th>Required:</th>
</tr>
</thead>
</table>
| - Driving gear is on the right.  
  - Possible gear sizes are 40, 24, 12, 8 tooth gears. | - Calculate the ratio.  
  - Write “Driver turns N\(X\), Follower turns M\(X\)” |

#### Example

1. **(# Teeth on driven gear)**

   $$\frac{40}{8} = \frac{5}{1}$$

   **(# Teeth on driving gear)**

   **Sample Answer:**
   - 5 : 1
   - Driver turns 5\(X\)
   - Follower turns 1\(X\)

2. **(# Teeth on driven gear)**

   **Sample Answer:**
   - Driver turns \(N\)\(X\)
   - Follower turns \(M\)\(X\)

3. **(# Teeth on driven gear)**

   **Sample Answer:**
   - Driver turns \(N\)\(X\)
   - Follower turns \(M\)\(X\)

4. **(# Teeth on driven gear)**

   **Sample Answer:**
   - Driver turns \(N\)\(X\)
   - Follower turns \(M\)\(X\)

5. **(# Teeth on driven gear)**

   **Sample Answer:**
   - Driver turns \(N\)\(X\)
   - Follower turns \(M\)\(X\)
**Gear Ratios Word Problems**

Instructions

Christine and Tim are designing robots with different gear ratios to complete various tasks. Use the formulas below to determine what gears and gear ratios they should use.

\[
\frac{\text{#Teeth on follower (output) gear}}{\text{#Teeth on driver (input) gear}} = \text{Gear Ratio}
\]

1. Christine and Tim want to design a robot with as high a gear ratio as possible in order to climb the greatest possible slope. They have 40, 24, 12, and 8 tooth gears available.
   - What gear should they choose as their output and input gears?
   - What would be the gear ratio of this robot?

2. Christine and Tim want to design with as low a gear ratio as possible so that it can be possible to reach a greatest speed. They have pulleys with diameters 5, 3, 1 cm available.
   - What diameter pulley should they choose as their output and input pulley?
   - What would be the gear ratio of this robot?

3. Christine and Tim want to design a robot with a gear ratio of 2:1, using a 16-tooth output gear. There are 24, 16, 12 and 8 tooth gears available. What gear should they choose as their input gear?

4. Christine and Tim want to design a robot with a gear ratio of 0.6, using 40-tooth driving gear. They have 24-, 12-, 8-, and 6- tooth gears available. What gear should they choose to be the follower gear.


**Instructions**

For each set of gears first determine the gear ratio, and reduce that to its smallest proportion. Use the reduced gear ratio for problems A and B. In problem A assume that the driven gear moves at a constant 100 RPM. At what speed is the driving gear rotating? In problem B assume that the driven gear moves at 250 RPM. At what speed is the driving gear rotating?

*Note: The driving gear is always on the right. Possible gear sizes are 40, 24, 14 and 8 tooth gears.*

**Use the formulas below:**

1. \[
\frac{\text{# Teeth on driven gear}}{\text{# Teeth on driving gear}} = \text{Gear Ratio}
\]

2. \[(\text{Gear Ratio}) \times (\text{Speed of Driven Gear}) = (\text{Speed of Driving Gear})\]

---

**Example 1**

1. \[
\frac{40}{8} = \frac{5}{1}
\]

\[
\begin{align*}
\text{A.} & \quad \frac{5}{1} \\
\text{B.} & \quad \frac{5}{1}
\end{align*}
\]

\[
\begin{align*}
\times 100 \text{ RPM} & = 500 \text{ RPM} \\
\text{(Speed of Driven)} & = \text{(Speed of Driving)} \\
\times 250 \text{ RPM} & = 1250 \text{ RPM}
\end{align*}
\]

**Example 2**

1. \[
\frac{\text{# Teeth on driven gear}}{\text{# Teeth on driving gear}}
\]

\[
\begin{align*}
\text{A.} & \quad \quad \\
\text{B.} & \quad \quad 
\end{align*}
\]

\[
\begin{align*}
\times 100 \text{ RPM} & = \quad \quad \\
\text{(Speed of Driven)} & = \quad \quad \\
\times 250 \text{ RPM} & = \quad \quad 
\end{align*}
\]

**Example 3**

1. \[
\frac{\text{# Teeth on driven gear}}{\text{# Teeth on driving gear}}
\]

\[
\begin{align*}
\text{A.} & \quad \quad \\
\text{B.} & \quad \quad 
\end{align*}
\]

\[
\begin{align*}
\times 100 \text{ RPM} & = \quad \quad \\
\text{(Speed of Driven)} & = \quad \quad \\
\times 250 \text{ RPM} & = \quad \quad 
\end{align*}
\]

**Example 4**

1. \[
\frac{\text{# Teeth on driven gear}}{\text{# Teeth on driving gear}}
\]

\[
\begin{align*}
\text{A.} & \quad \quad \\
\text{B.} & \quad \quad 
\end{align*}
\]

\[
\begin{align*}
\times 100 \text{ RPM} & = \quad \quad \\
\text{(Speed of Driven)} & = \quad \quad \\
\times 250 \text{ RPM} & = \quad \quad 
\end{align*}
\]
Geared Ratio and Speed Problem Solving

Instructions
Use the formulas and pictures below to answer the following questions
Note: The driving gear is always on the right. Possible gear sizes are 40, 24, 14 and 8 tooth gears.

The formulas are:

1. \[
\frac{\text{(# Teeth on driven gear)}}{\text{(# Teeth on driving gear)}} = \text{Gear Ratio}
\]

2. \[
(Gear Ratio) \times \text{(Speed of driven gear)} = \text{(Speed of driving gear)}
\]

3. \[
\frac{\text{Wheel Circumference}}{\text{Revolutions}} \times 1 = \text{Distance}
\]

Leave your answer in fraction.

1. Assuming the circumference of the wheel and the RPM of the motor are exactly the same for all experiments, which gear would create the most speed, A, B, or C? Why?

2. If the driving gear is moving at 100RPM, how fast will the driven gear move for diagram A, B, and C?

3. The driving gear moves at 100 RPM and the circumference of the wheel is 12cm. How far will the robot moves in 2 minutes for diagram A, B, and C?

4. The driving gear moves at 200 RPM and the diameter of the wheel is 6cm. How far will the robot moves in 7 minutes for diagram A, B, and C?

5. The driving gear moves at 100 RPM and the radius of the wheel is 6cm. How far will the robot moves in 2 minutes for diagram A, B, and C?
**What is the big deal about gear ratio?**

- Strength ...
- Speed ...
- Distance, of course.
- One more... ________________________

**Let's do a simple experiment.** Build a robot (use a vehicle chassis, not walking chassis). Make sure you can easily change the gear ratio at ease.

Hypothesis: _Gearing down will yield high accuracy_

Independent Variable: Time in seconds

Dependent Variable: Distance in cm

Control Variable: Gear Ratios

**Test 1**

a. Create a configuration with the first gear ratio.
b. Program it to run for 5 seconds.
c. Run it for 3 times, and measure the distance it travels for each time.
d. Write down the error margin for each run.e. Take an average.f. Plot it on the graph.

**Test 2**

a. Modify the gear ratio to gearing down.b. Program it to run for 5 seconds.c. Run it for 3 times, and measure the distance it travels for each time.d. Write down the error margin for each run.e. Take an average.f. Plot it on the graph.

**Test 3**

a. Modify the gear ratio again to gear it down.b. Program it to run for 5 seconds.c. Run it for 3 times, and measure the distance it travels for each time.d. Write down the error margin for each run.e. Take an average.f. Plot it on the graph.

For example: Gear Ratio : 5: 1. Duration: 5 seconds

<table>
<thead>
<tr>
<th># of trial</th>
<th>Error Margin (after calculating the average)</th>
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<tbody>
<tr>
<td>1\textsuperscript{st}: 13 cm</td>
<td>1</td>
</tr>
<tr>
<td>2\textsuperscript{nd}: 12 cm</td>
<td>0</td>
</tr>
<tr>
<td>3\textsuperscript{rd}: 11 cm</td>
<td>-1</td>
</tr>
<tr>
<td>Overall Average : 12 cm</td>
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Wheels Diameter vs. Distance Traveled

Learn about the relationships between wheel radius, diameter, circumference, revolutions and distance.

Convert Distance Traveled to Rotation counts.

Based on the information provided about the wheels shown on these pages, calculate how far they will travel.

Steps:

a. 1 rotation sensor rotation = 16 counts (note: this can vary, NXT ➔ 360 counts)
b. 1 tire revolution = 16 counts
c. 16 counts = 10* PI cm * G.R.
d. 1 count = (10* PI ) / 16 cm or
e. 1 cm = 16 / (10*PI) count

Sample Question 1:

Let diameter of the tire = 10cm GR = 1:1

How many counts should the robot need to travel 20cm?

Steps:

a. ∵ G.R. = 1/1
b. i.e Tire turns 1X = Rotation sensor turns 1X
c. ∵ 1 rotation = 16 counts
d. ∴ Turn turns * G/R = 16 counts
e. Dia cm * PI * G.R. = 16 counts
f. 1 cm = 16 / (dia*PI * G.R. ) counts
g. 1 cm = 16 / (10PI) counts
h. For 20cm: 20 cm = 32 / PI counts
Sample question 2:

Let diameter of the tire = 10cm

GR = 16:40 = 2:5

How many counts should the robot need to travel 200cm?

Let diameter of the tire = 10cm

GR = 16:40 = 2:5

How many counts should the robot need to travel 20cm?

Steps:

a. \( \therefore \) G.R. = 2/5

b. i.e TIre turns 2/5X = Rotation sensor turns 1X

c. \( \therefore \) 1 rotation = 16 counts

d. \( \therefore \) Turn turns * G.R. = 16 counts

e. Dia cm * PI * G.R. = 16 counts

f. 1 cm = 16 / (dia*PI * G.R.) counts

g. 1 cm = 16 / (10*PI * 2/5) counts

h. For 20cm: 20 cm = 20 * 16 / 4 *PI counts

i. \( \approx \) 80 / PI counts

Program for simple going forward

Given:
1 rotation sensor revolution = 16 counts
GR = 3/5
Tire diameter = 5.6
PI = 3.14159

Let say your robot needs to run 25 cm.

1) calculate the gear ratio
2) create a variable "rCounts" = # of rotation counts required to run for 20 cm
3) convert the distance in cm to # rotation counts
i.e. rCounts = 16 * K counts / (tire diameter * PI * gr)

\[ rCounts = 25 \times 16 / (5.6 \times 3.1415 \times 3/5) \]
**Wheels Diameter to Distance Exercise**

Note: You should leave the answer in expression with PI. For example, if the answer is 3PI, you should leave your answer as 3PI, instead of approx.. 9.42.

1. Ashley and John recorded that the wheel diameter was 10cm and that the robot was programmed to move 2 wheel revolutions. How far did their robot move in cm?

2. Ashley and John recorded that their robot moved 120 cm and that the robot was programmed to move 4 wheel revolutions. What was the diameter of the wheel in inches?

3. Ashley and John recorded that their robot moved 250 cm and that the wheel diameter was 5 cm. How many wheel revolutions was the robot programmed to complete?

4. Tire diameter = 10cm  
   GR = 1:1  
   1Rotation Revolution= 16 counts  
   Distance to travel = 200cm  
   How many counts should the robot need to travel?

5. Tire diameter = 10cm  
   GR = 2:1  
   1Rotation Revolution= 16 counts  
   Distance to travel = 200cm  
   How many counts should the robot need to travel?

6. Tire diameter = 10cm  
   GR = 1:2  
   1Rotation Revolution= 16 counts  
   Distance to travel = 200cm  
   How many counts should the robot need to travel?

7. Tire diameter = 10cm  
   GR = 2:5  
   1Rotation Revolution= 16 counts  
   Distance to travel = 200cm  
   How many counts should the robot need to travel?

8. Tire diameter = 10cm  
   GR = 5:2  
   1Rotation Revolution= 16 counts  
   Distance to travel = 200cm  
   How many counts should the robot need to travel?
**CHASSIS TURNING vs. ROTATION SENSOR ROTATION**

1) Convert Chassis turning degrees to Distance Travel
2) Convert Distance Traveled to Rotation counts

Steps:

a. Calculate $1 \text{ cm} = ?$ rotation counts...

   i.e. $1 \text{ cm} = \frac{16}{\text{dia} \times \pi \times \text{GR}}$ counts

b. Calculate the distance traveled for turning.

   e.g. let $W =$ WheelBase

   one full revolution $= W \times \pi \text{ cm}$

   $W \times \pi \text{ cm} = 360$ degrees

   $1 \text{ cm} = \frac{360}{W \times \pi}$ degrees

   $\therefore 1 \text{ cm} = \frac{16}{\text{dia} \times \pi \times \text{GR}}$ counts

   $\therefore \frac{360}{W \times \pi}$ degrees $= \frac{16}{\text{dia} \times \pi \times \text{GR}}$ counts

   $1$ degree $= \frac{16 \text{ counts}}{\text{dia} \times \pi \times \text{GR}} \times \frac{W \times \pi}{360}$

   $= \frac{16 \text{ counts}}{\text{dia} \times \pi \times \text{GR}} \times \frac{W}{360}$

   $= \frac{16 \text{ counts}}{\text{dia} \times \text{GR}} \times \frac{W}{360}$

**Formula:** $1$ degree $= \frac{16 \times W \text{ counts}}{\text{dia} \times \text{GR} \times 360}$
Sample Problem:

Given:  
\[
\begin{align*}
W &= 21\text{cm} \\
\text{Dia} &= 7\text{cm} \\
\text{GR} &= 1 : 5
\end{align*}
\]

1 degree = \(\frac{16 \times 21 \text{ counts}}{7 \times \left(\frac{1}{5}\right) \times 360}\)

= \(\frac{16 \times 21 \times 5 \text{ counts}}{7 \times 1 \times 360}\)

= \(\frac{2 \times 3 \times 5 \text{ counts}}{1 \times 1 \times 45}\)

= \(\frac{2 \times 3 \times 1 \text{ counts}}{1 \times 1 \times 9}\)

= \(\frac{2 \times 1 \times 1 \text{ counts}}{1 \times 1 \times 3}\)

90 degrees = \(90 \times \frac{2 \text{ counts}}{3}\)

= 60 counts